PART : PHYSICS

1. Radius of electron in ground state of hydrogen is a_0, and radius of electron in He^+ ion in 3rd excited state is a, then $\frac{a_0}{a}$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{16}$ (4) $\frac{1}{8}$

Ans. **(4)**

Sol. $a = a_0 \frac{n^2}{Z}$

$\frac{Z}{n^2} = \frac{a_0}{a}$

$\frac{2}{16} = \frac{a_0}{a}$

$\frac{a_0}{a} = \frac{1}{8}$

2. Electric flux ϕ is related with linear charge density λ and surface charge density σ as $\phi = \alpha\lambda + \beta\sigma$ where α and β are of appropriate dimension then dimension of (β/α) is:

(1) Displacement (2) Area (3) Electric field (4) Velocity

Ans. **(1)**

Sol. $\phi = \alpha\lambda + \beta\sigma$

$\alpha\lambda = \beta\sigma$

$\frac{\beta}{\alpha} = \frac{\lambda}{\sigma} = \frac{Q/L}{Q/L^2} = \frac{Q}{L} \times \frac{L^2}{Q}$

$= L$ (Length)

3. The displacement of a particle as function of time is $x(t) = A\sin(t) + B\cos^2(t) + Ct^2 + D$.

Find dimension of $\left[\frac{ABC}{D}\right]$

(1) L^2 (2) L^2T^{-2} (3) LT^{-2} (4) L^3T

Ans. **(2)**

Sol. Dimension $\rightarrow$ A $\rightarrow$ [L]

Dimension $\rightarrow$ B $\rightarrow$ [L]

$C[T^2] = [L]$

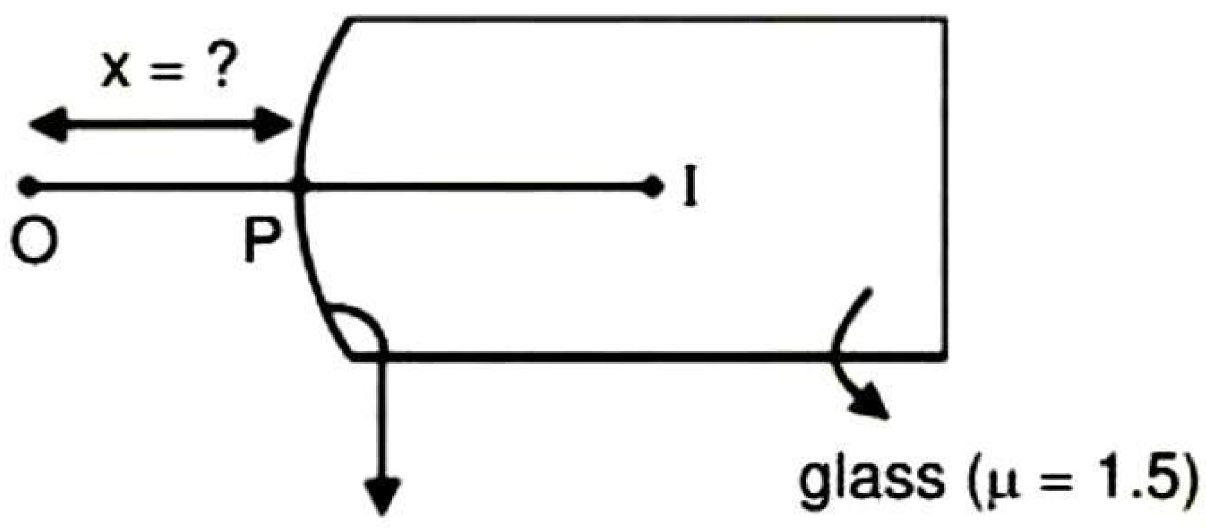
Dimension $\rightarrow$ C $\rightarrow$ $[LT^{-2}]$

Dimension $\rightarrow$ D $\rightarrow$ [L]

So, $\frac{ABC}{D} \Rightarrow \frac{[L][L][LT^{-2}]}{[L]}$

Dimension = $[L^2T^{-2}]$

4. A small point object is placed at some distance from a convex spherical surface of radius of curvature R and refractive index 1.5 as shown in the diagram. It was found that image is formed at equal distance from P. If the distance at which object is placed from point P is x then find x.



(1) 1.5 R (2) 2 R (3) 3 R (4) 5 R

Ans. (4)

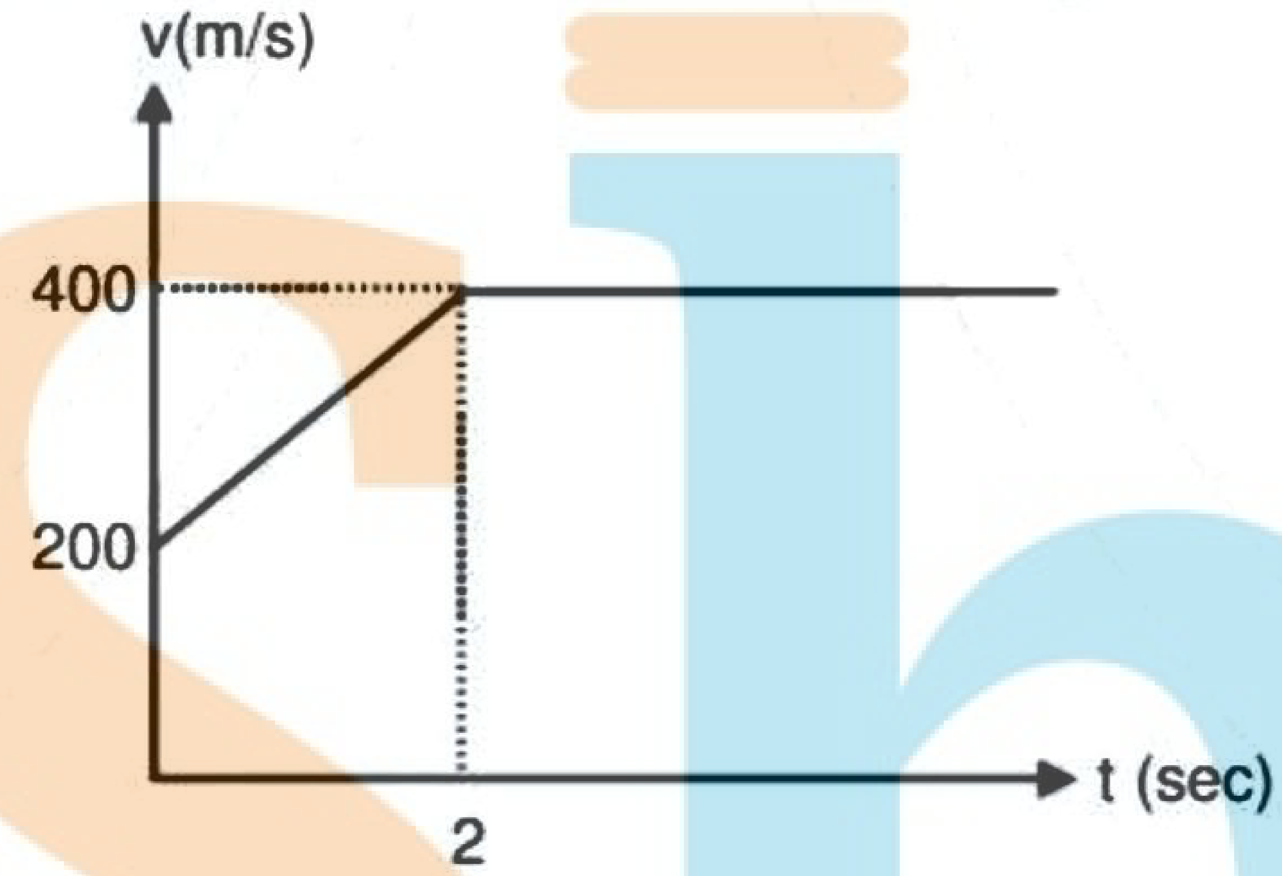
Sol. $\frac{\mu_2}{v} - \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.5}{v} + \frac{1}{v} = \frac{0.5}{R}$

$= \frac{2.5}{v} = \frac{0.5}{R}$

$v = 5R$

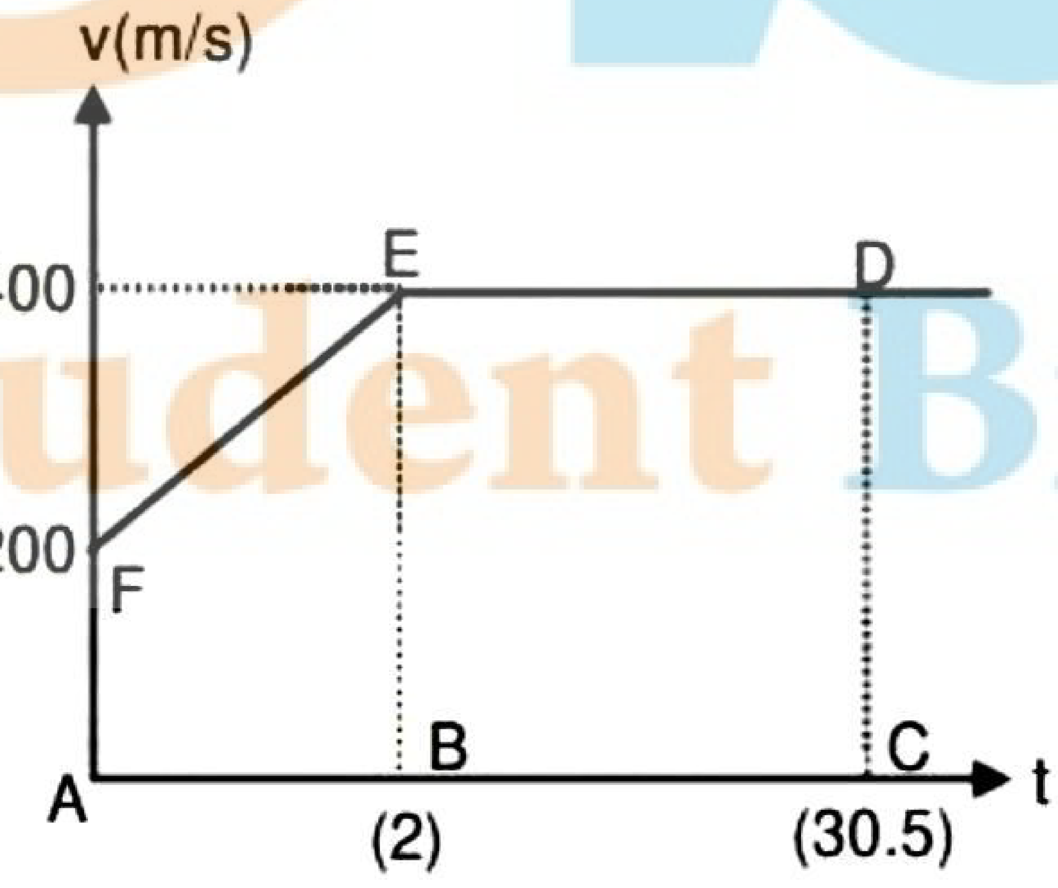
5. For given velocity - time (v-t) graph. Find the distance travelled upto 30.5 sec



(1) 10 Km (2) 11 Km (3) 12 Km (4) 13 Km

Ans. (3)

Sol. Distance = $\int_0^{30.5} |\vec{v}|dt$ = Area under v-t graph with +ve sign.



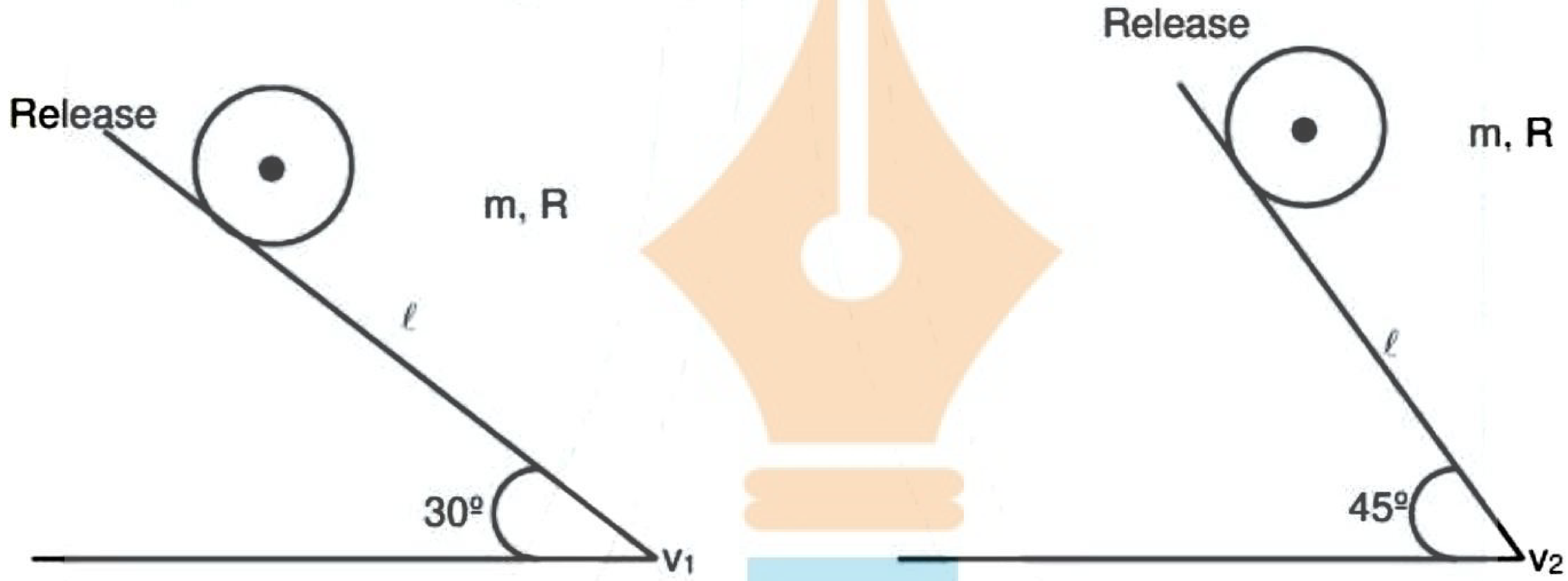
$\Rightarrow$ Area (ABEF) + Area (BCDE)

$\Rightarrow \frac{1}{2} \times (200 + 400) \times 2 + 400\ (30.5 - 2)$

$= 600 + \frac{400 \times 2865}{10}$

$= 600 + 11400 = 12000$ m = 12 Km

6. Two identical ball of mass m and radius R are released from rest on two inclined planes of length ℓ as shown in diagram. If balls are rolling without sliding then find the ratio of the square of the speed $(v_1^2 : v_2^2)$ with which they will reach on the ground.



(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. (2)

Sol. H.S

$$a_1 = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$$

$$v_1^2 : v_2^2$$

$$a_1 = \frac{g \times \sin 30}{1 + \frac{2/3mR^2}{mR^2}}$$

$$a_2 = \frac{g\sin 45}{1 + \frac{2/3mR^2}{mR^2}}$$

$$a_1 = \frac{g/2}{5/3} = \frac{3g}{10}$$

$$a_2 = \frac{g/\sqrt{5}}{5/3} = \frac{3g}{5\sqrt{2}}$$

$$\frac{v_1^2}{v_2^2} = \frac{2 \times a_1 \times l}{2a_2 \times 1} = \frac{3g/10}{7g/5\sqrt{2}} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\frac{v_1^2}{v_2^2} = \frac{1}{\sqrt{2}}}$$

7. **Statement-I :** Hot water moves faster than cold water.
Statement-II : Soap water have higher surface tension than fresh water.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

Ans. **(3)**

Sol. **Statement –I:** Due to increase in temperature Viscosity decreases and K.E. increases so hot water moves faster so it is true
Statement-II : Soap water have Lower surface tension than fresh water.

8. If force $\vec{F} = x^2y\hat{i} + y^2\hat{j}$ is acting on a particle along the line y = x for displacement from A(0, 0) to B(4, 4). Find work done by the force

(1) $\frac{256}{3}$ J (2) 64 J (3) $\frac{64}{3}$ J (4) 256 J

Ans. **(1)**

Sol. Given force $\vec{F} = x^2y\hat{i} + y^2\hat{j}$

Work done $W = \int \vec{F}.d\vec{r} = \int (x^2y\hat{i} + y^2\hat{j}).(dx\hat{i} + dy\hat{j})$

$$= \int_0^4 x^2ydx + \int_0^4 y^2dy$$

$$= \int_0^4 x^3dx + \int_0^4 y^2dy = \frac{256}{3} \text{ J}$$

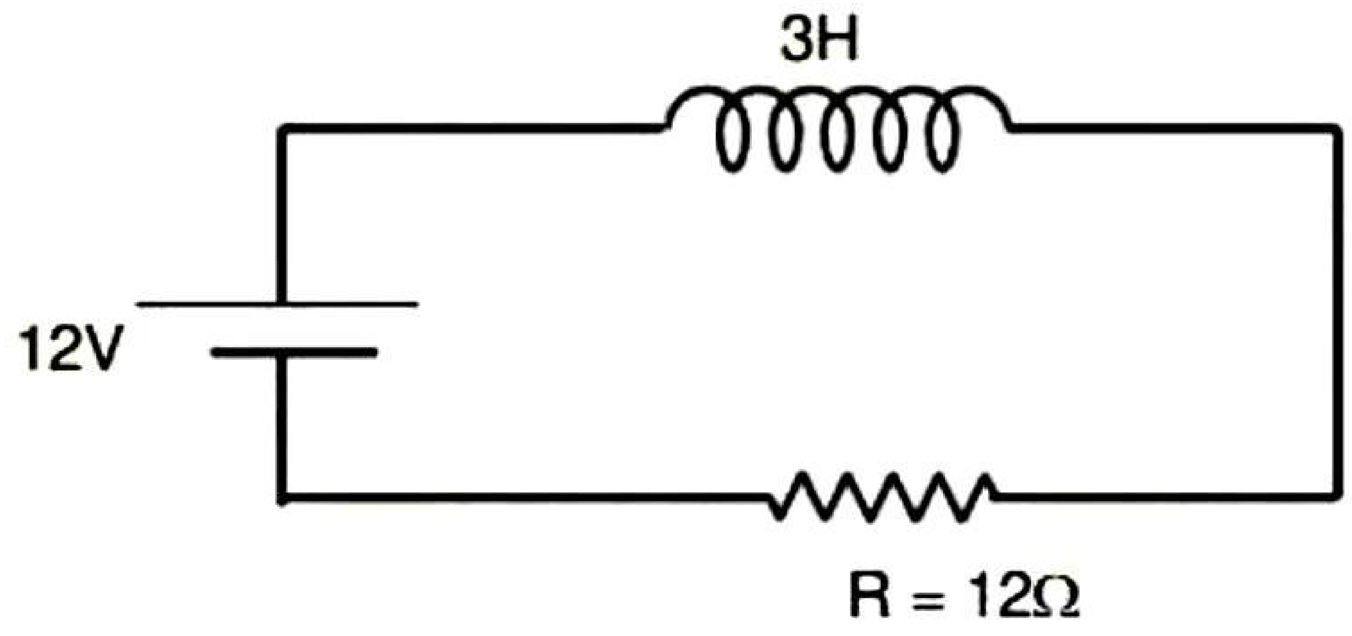
9. Match the column

Column-I	**Column-II**
(P) When volume change is zero	(i) Isochoric process
(Q) When pressure is constant	(ii) Adiabatic process
(C) When no heat is exchanged	(iii) Isobaric process
(D) Work done by the gas is equal to heat given to the gas	(iv) Isothermal process

(1) P → (i) ,Q → (iii), R → (ii), S → (iv)
(2) P → (ii) ,Q → (iii), R → (i), S → (iv)
(3) P → (iii) ,Q → (i), R → (ii), S → (iv)
(4) P → (iv) ,Q → (iii), R → (ii), S → (i)

Ans. **(1)**

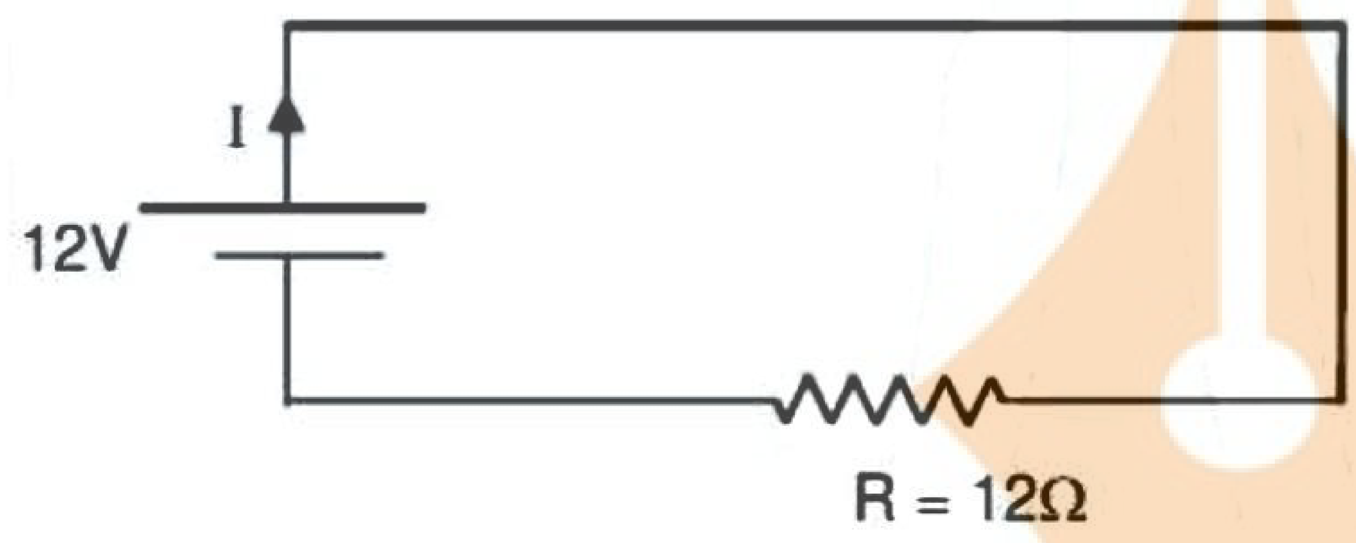
10. In the given DC circuit, find the current through R = 12Ω in steady state



(1) 2A (2) 3A (3) 1A (4) 4A

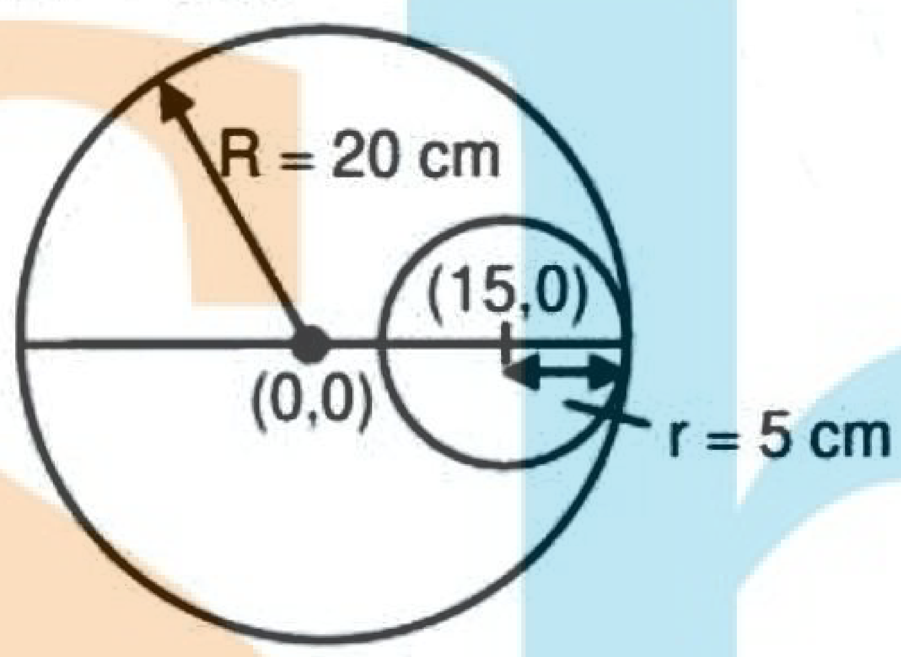
Ans. (3)

Sol. In steady state inductor is short circuited



$I = \frac{12}{12} = 1\ A$

11. Find the position of centre of mass of uniform dics of radius 20 cm with respect to previous origin if small disc of radius 5 cm cut from the disc



(1) (–2, 0) (2) (1, 0) (3) (–1, 0) (4) (–5, 0)

Ans. (3)

Sol. Let surface mass density = σ

M_1 (mass of disc before removal = $\sigma(\pi R^2)$

M_2 (mass of smaller disc = $\sigma(\pi r^2)$

COM of smaller disc = (15, 0)

COM of disc after removal of disc

$$x_{COM} = \frac{M_1x_1 - M_2x_2}{M_1 - M_2} = \frac{\sigma(\pi R^2 \times 0) - (\sigma\pi r^2) \times 15}{\sigma\pi(R^2 - r^2)}$$

$$x_{COM} = \frac{-15r^2}{R^2 - r^2} = \frac{-15 \times 5 \times 5}{(20)^2 - (5)^2} = \frac{-15 \times 5 \times 5}{25 \times 15} = -1$$

$y_{COM} = 0$

COM (–1, 0)

12. The ratio of electric force to gravitational force between two particles having charges q_1, q_2 and masses m_1 and m_2 respectively (where symbols have their usual meanings).

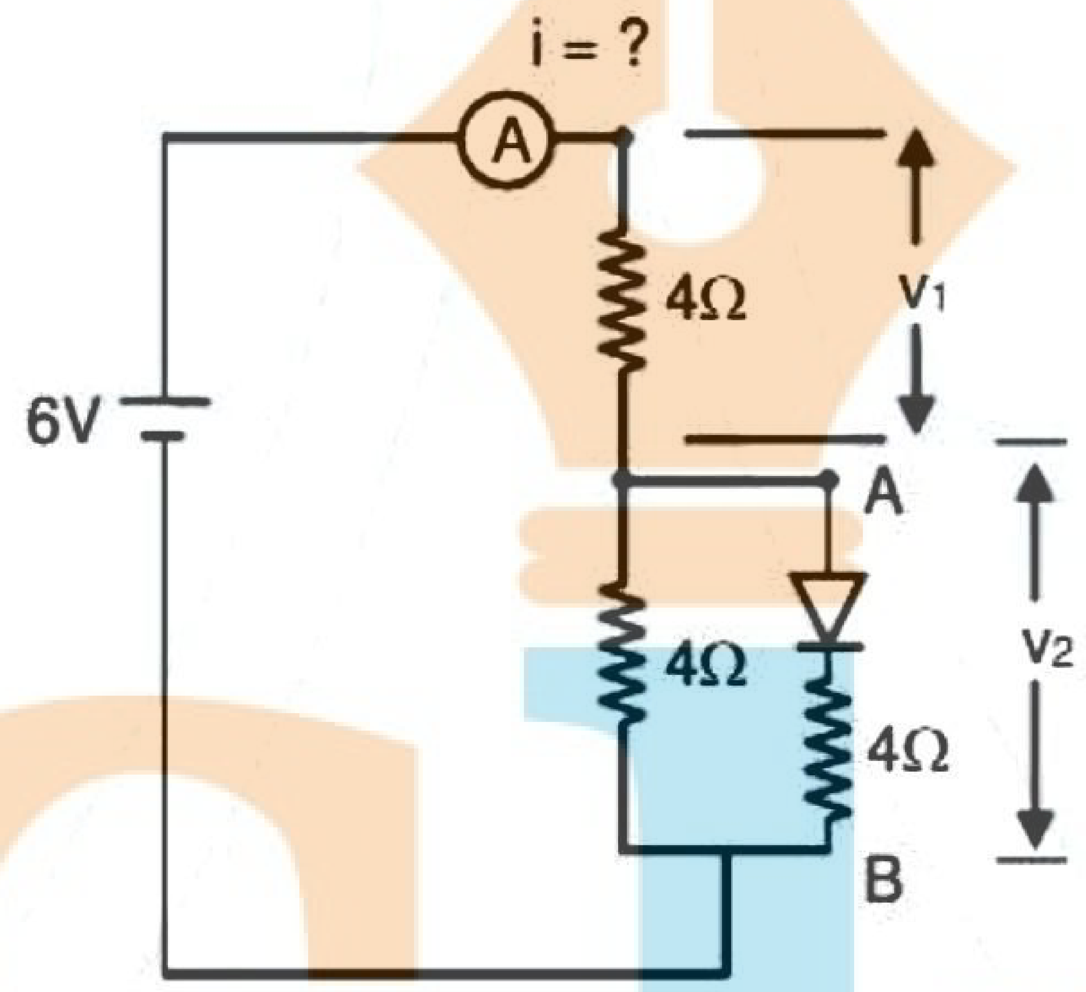
(1) $\frac{4\pi\varepsilon_0 m_1 m_2 G}{q_1 q_2}$ (2) $\frac{4\pi\varepsilon_0 G m_1 m_2}{q_1 q_2 r^4}$

(3) $\frac{q_1 q_2 r^4}{4\pi\varepsilon_0 G m_1 m_2}$ (4) $\frac{q_1 q_2}{4\pi\varepsilon_0 G m_1 m_2}$

Ans. **(4)**

Sol. $\frac{F_E}{F_G} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}{G\frac{m_1 m_2}{r^2}} = \frac{q_1 q_2}{4\pi\varepsilon_0 G m_1 m_2}$

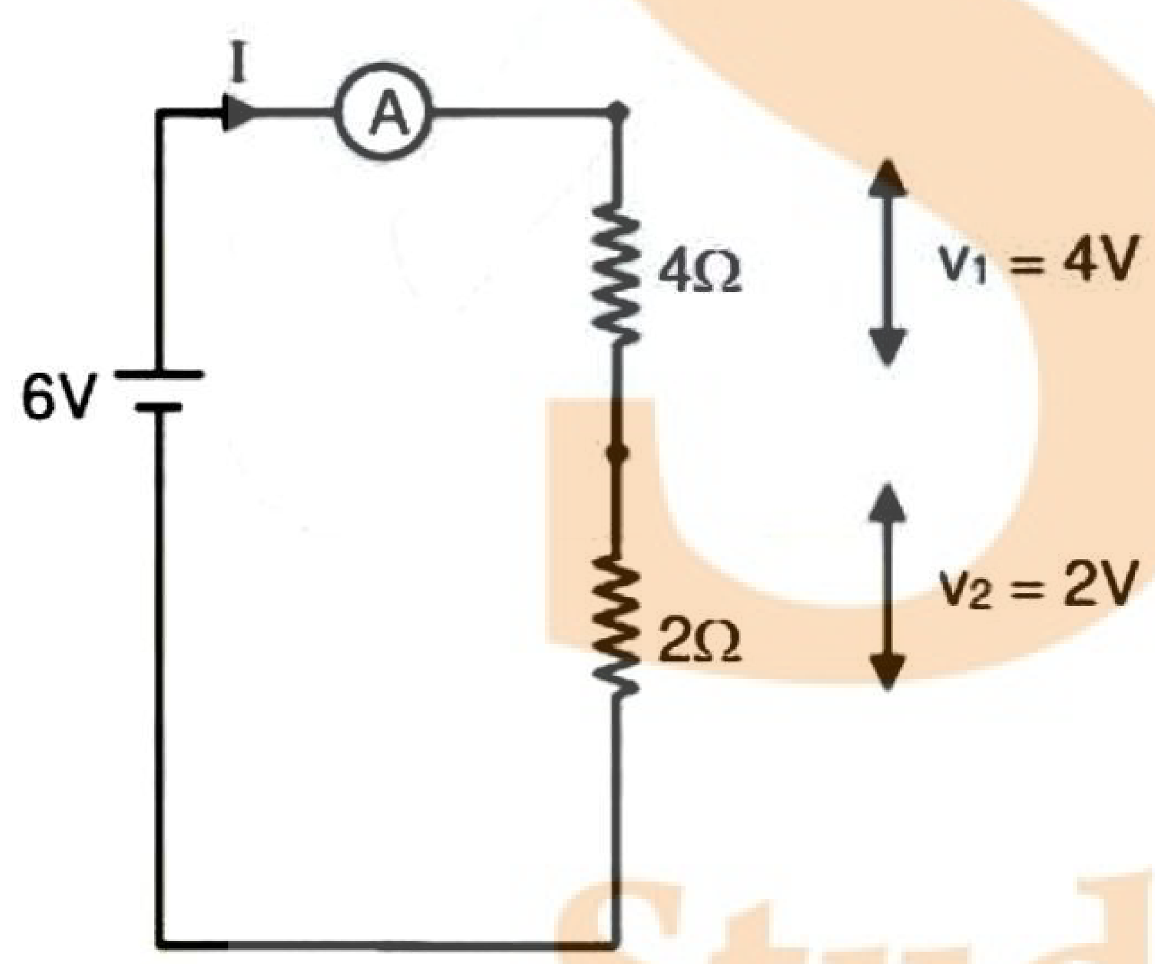
13. Which one is the correct option for given circuit for i, v_1, and v_2



(1) 1 Amp. 2V, 4V (2) 1 Amp. 4V, 2V (3) 2 Amp. 4V, 2V (4) 0.1 Amp. 2V, 4V

Ans. **(2)**

Sol.



$v_4 = \frac{4}{8}\times 6 = \frac{1}{2}\times 6 = 3$ volt

Diode on

$I = \frac{6}{6} = 1$ A

14. Self inductance depends on

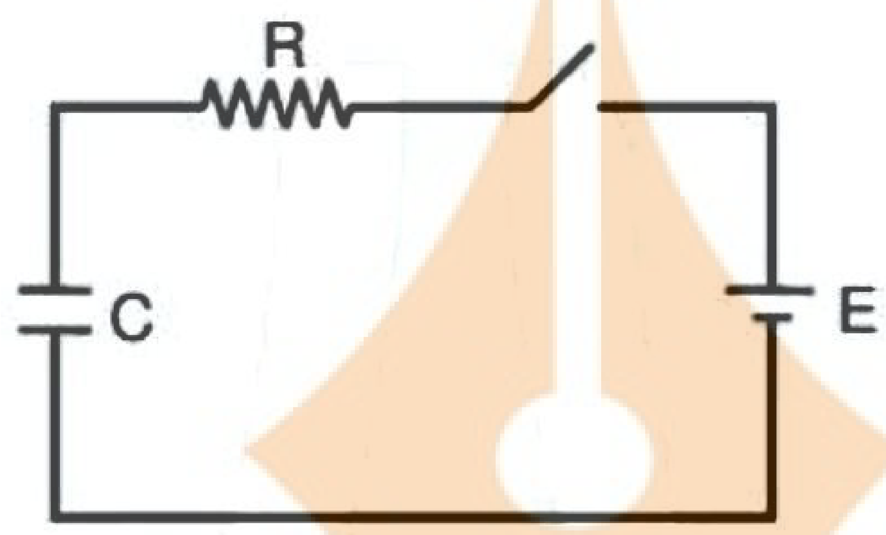
(1) only on geometry
(2) only on medium property
(3) Geometry and medium property
(4) value of current through inductor

Ans. **(3)**

Sol. Self-inductance = $\underset{\text{Medium}}{\underset{\downarrow}{\mu_r\mu_0}}\ \ N_2\ \ \underset{\text{Geometry}}{\underset{\downarrow}{Al}}$

So, depends on Geometry and medium.

15. The key shown in the circuit is closed at t = 0.
Choose the incorrect option regarding the condition at t = 0



(1) Current in the circuit is zero
(2) Voltage across the capacitor is minimum
(3) Current in the circuit is maximum
(4) Voltage across resistance is maximum

Ans. **(1)**

Sol. $i(t) = \frac{E}{R} e^{-t/RC}$

$q(t) = CE(1 - e^{-t/RC})$

$t = 0, \quad i(t = 0) = \frac{E}{R}$

option (1) $\rightarrow$ incorrect t

$t = 0$, $q(t = 0) = 0$

$v_C = \frac{q}{c} = 0$ (minimum voltage)

options (ii) $\rightarrow$ correct

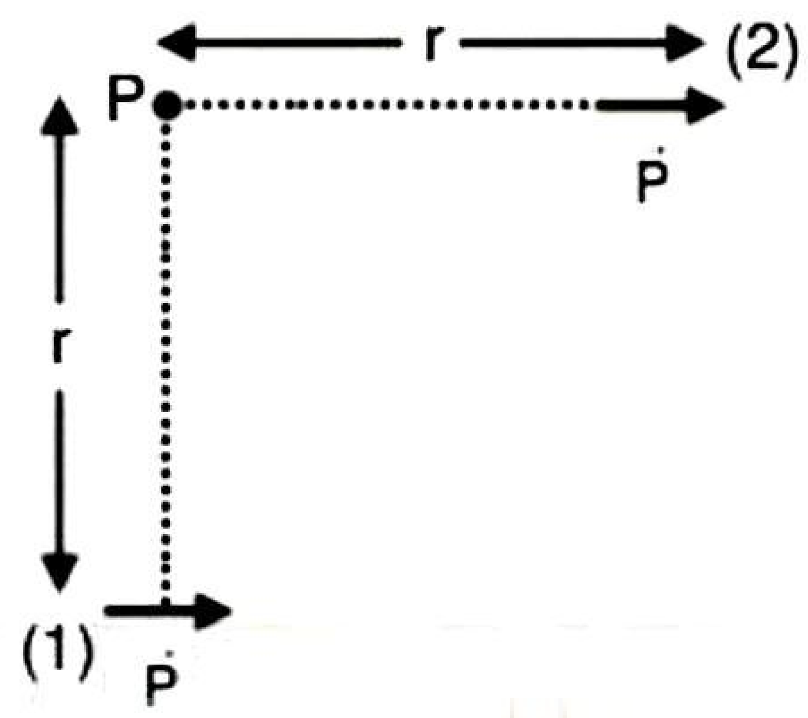
$i(t = 0) = \frac{E}{R}$ (maximum correct)

options (iii) $\rightarrow$ correct

$v_R = iR = \frac{E}{R} \times R = E \quad (t = 0)$

option (iv) $\rightarrow$ correct

16. If two dipoles of dipole moment p are placed as shown in figure. Find the net electric force which feels by a point unit charge at point P?



(1) $\frac{2KP}{r^3}$ (2) $\frac{KP}{2r^3}$ (3) $\frac{\sqrt{2}KP}{r^3}$ (4) $\frac{KP}{r^3}$

Ans. **(4)**

Sol. Net electric force at point P

$\vec{F} = \vec{F}_1 + \vec{F}_2$

$= \frac{kP}{r^3}(-\hat{i}) + \frac{2KP}{r^3}(+\hat{i})$

$\vec{F} = \frac{KP}{r^3}\hat{i}$

17. Adiabatic constant of a gas is $\frac{3}{2}$. If volume of gas initially at 0ºC is reduced to one fourth of the original volume then new temperature is

(1) 0K (2) 273 K (3) 546ºC (4) 546 K

Ans. **(4)**

Sol. PV^y = constant

TV^{y-1} = constant

$T_1V_1^{y-1} = T_2V_2^{y-1}$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{y-1} = 273\left(\frac{4V_1}{V_1}\right)^{\frac{3}{2}-1}$

$T_2 = 273 \times 2 = 546$ K

18. Given a convex lens of refractive index μ_2 in a liquid of refractive index μ_1, $\mu_1 < \mu_2$ having radii of curvature R_1, R_2 then R_2 surface a silver polished. Where should an object be placed on the optical axis so that the real and inverted image is formed at the same place

(1) $\frac{(\mu_2+\mu_1)|R_1|}{(\mu_2-\mu_1)}$

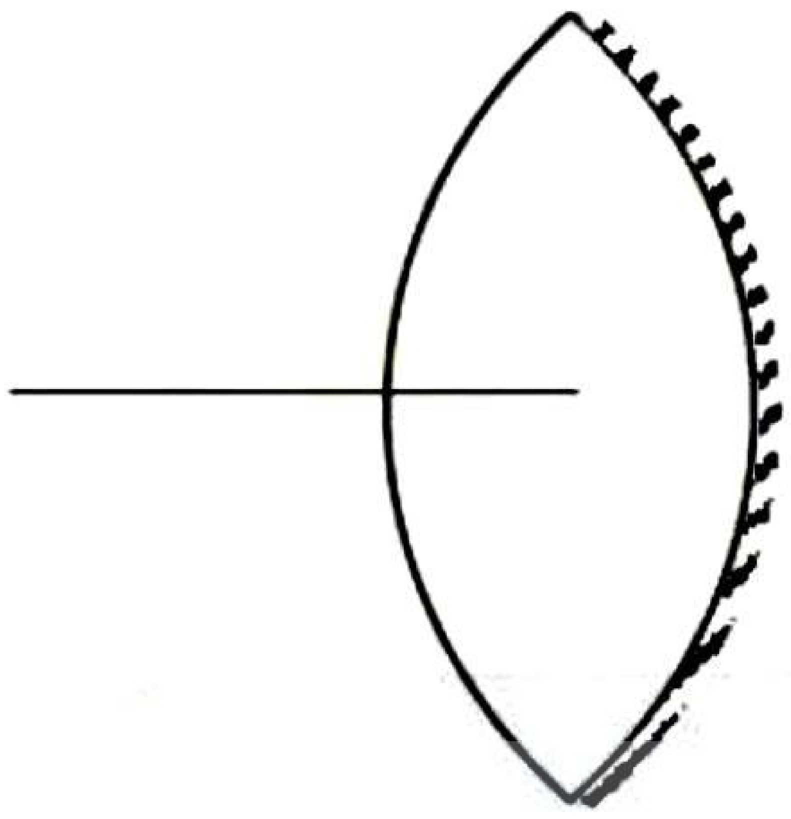
(2) $\frac{\mu_1|R_1||R_2|}{\mu_2(|R_1|+|R_2|-\mu_1|R_2|)}$

(3) $\frac{\mu_1|R_1||R_2|}{\mu_2(2|R_1|+|R_2|-\mu_1|R_2|)}$

(4) $\frac{\mu_1|R_1|-|R_2|}{\mu_2(|R_1|+|R_2|-\mu_1\sqrt{|R_1|-|R_2|})}$

Ans. **(2)**

Sol.



$$\frac{1}{f_C} = -\frac{-2}{f_L} + \frac{1}{f_M}$$

$$\Rightarrow \frac{1}{f_C} = -2\left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{2}{R}$$

$$\Rightarrow \frac{1}{f_C} = \frac{-2(\mu_2 - \mu_1)}{\mu_1}\frac{(R_1 + R_2)}{R_1R_2} - \frac{2}{R_2} \times \frac{\mu_1R_1}{\mu_1R_1}$$

$$= \frac{-\mu_1R_1R_2}{(\mu_2 - \mu_1)(R_1 + R_2) + \mu_1R_1} = \frac{-\mu_1R_1R_2}{\mu_2(R_1 + R_2) - \mu_1R_2}$$

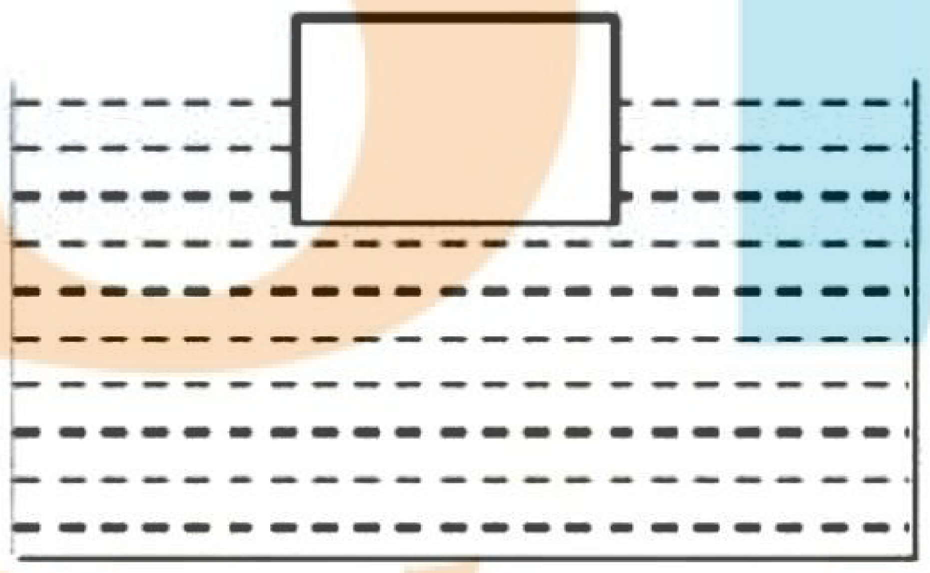
19. What is the dimensional formula of torsional constant ?

(1) $[ML^2T^{-2}]$ (2) $[ML^3T^2]$ (3) $[M^oLT^2]$ (4) $[M^2L^1T^2]$

Ans. **(1)**

Sol. $[ML^2T^{-2}]$

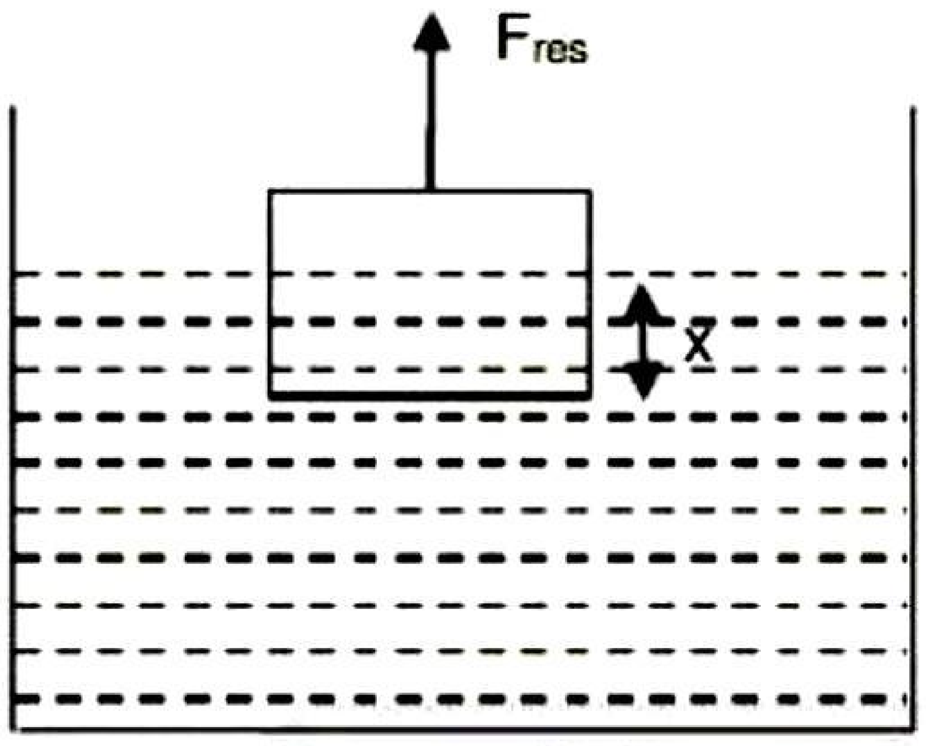
20. Find the time period of a cube of side length 10 cm and mass 10 gm oscillating in water (Density of water $= 10^3$ kg/m^3 and $g = 10$ m/s^2)



(1) $\frac{\pi}{25}$ second (2) $\frac{\pi}{50}$ seconds (3) $\frac{\pi}{100}$ second (4) $\frac{2\pi}{25}$ second

Ans. **(2)**

Sol.



$F_{res} = -(V_{in})\rho_w g = -(Ax)\rho_w g$

$-(Ax)\rho_w g = ma$

$-Ax\rho_w g = m(\omega^2 x)$

$\omega^2 = -\frac{A\rho_\omega}{m} g$

$\Rightarrow \omega = \frac{2\pi}{T} = \sqrt{\frac{A\rho_w g}{m}}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{A\rho_\omega g}}$

$\Rightarrow T = 2\pi\sqrt{\frac{10\times10^{-3}}{10\times10\times10^{-4}\times10^{-3}\times10}}$

$T = 2\pi\sqrt{\frac{10^{-3}}{10}} = 2\pi\sqrt{10^{-4}}$

$= 2\pi \times 10^{-2}$

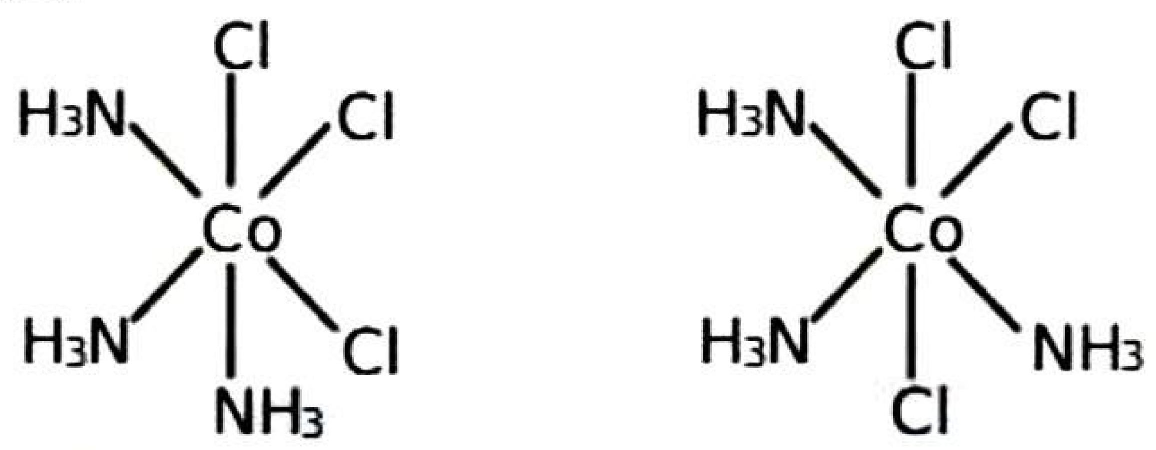
PART : CHEMISTRY

1. Which complex will show facial and meridional form-

(1) $[Co(NH_3)_6]$ (2) $[Co(NH_3)_5Cl]$ (3) $[Co(NH_3)_4Cl_2]$ (4) $[Co(NH_3)Cl_3]$

Ans. **(4)**

Sol.



Facial form Meridional form

2. The depression in freezing point of 0.1 molal solution is 0.558, then complex will be-

(1) $[Co(NH_3)_6Cl_2]$ (2) $[Co(NH_3)_5Cl]Cl$ (3) $[Co(NH_3)_4Cl_2]$ (4) $[Co(NH_3)Cl_3]$

Ans. **(1)**

Sol. $\Delta T_f = i.K_f.m$

$0.558 = i \times 1.86 \times 0.1$

$i = \frac{0.558}{1.86 \times 0.1} = 3$

3. Which of the following element does not lie in same period.

(1) Osmium (2) Iridium (3) Palladium (4) Platinum

Ans. **(3)**

Sol.

4. Which of the following pair of ions are same coloured ?

(1) Ti^{4+}, V^{3+}, Sc^{3+} (2) Cr^{2+}, Cu^{2+}, V^{4+} (3) Cr^{3+}, Ni^{2+}, V^{4+} (4) $Mn^{3+}, Fe^{2+}, Zn^{2+}$

Ans. **(2)**

Sol. Cr^{2+}, Cu^{2+}, V^{4+} (Blue)

5. Find ΔG of reaction at 298 K

$N_2O_4(g) \rightleftharpoons 2NO_2$;

$\Delta H = +50$ kJ/mol & $\Delta S = 5$ J/mol-k

Ans. **(48.5)**

Sol. $\because \Delta G = \Delta H - T\Delta S$

$= 50$ kJ/mol $- 298 \times \frac{5}{1000}$ kJ

$= 50 - 1.5$ kJ/mol

$= 48.5$ kJ/mol.

6. The pH of 0.1 M $C_2H_5NH_2$ solution is 9. if $K_b = 10^{-x}$ then find x.

Ans. **(9)**

Sol. $pOH = \frac{1}{2}(pK_b - \log C)$

$5 = \frac{1}{2}(pK_b - \log 10^{-1})$

$10 - 1 = pK_b = 9$

$k_b = 10^{-9} \Rightarrow x = 9$

7. Match the column-I and column-II

Column-I		Column-II	
(A)	Octet complete	(i)	BCl_3, $BeCl_2$
(B)	Octet expanded	(ii)	NO_2, NO
(C)	Octet incomplete	(iii)	CCl_4, CO_2
(D)	Odd electron	(iv)	H_2SO_4, PCl_5

(1) (A) → (iii) ; (B) → (iv) ; (C) → (i) ; (D) → (ii) (2) (A) → (iii) ; (B) → (i) ; (C) → (iv) ; (D) → (ii)
(3) (A) → (iv) ; (B) → (i) ; (C) → (ii) ; (D) → (iii) (4) (A) → (iv) ; (B) → (ii) ; (C) → (iii) ; (D) → (i)

Ans. **(1)**

8. If 10^{21} molecules are removed from x mg of $CO_2(g)$ then 2.8×10^{-3} mole are left. Calculate the value of x.

Ans. **(196.53)**

Sol. $(\text{mole})_i = \left(\frac{x \times 10^{-3}}{44}\right)$, $(\text{mole})_{\text{Removed}} = \left(\frac{10^{21}}{6 \times 10^{23}}\right) = \frac{1}{6} \times 10^{-2}$

$(\text{mole})_{\text{left}} = 2.8 \times 10^{-3}$

Now,
$(\text{mole})_i - (\text{mole})_{\text{Removed}} = (\text{mole})_{\text{left}}$

$$= \frac{x \times 10^{-3}}{44} - \frac{10^{21}}{6 \times 10^{23}} = 2.8 \times 10^{-3}$$

$$\frac{x \times 10^{-3}}{44} = 2.8 \times 10^{-3} + \frac{1}{6} \times 10^{-2} = \left(2.8 + \frac{10}{6}\right) \times 10^{-3}$$

$$\frac{x \times 10^{-3}}{44} = \left(\frac{16.8 + 10}{6}\right) \times 10^{-3}$$

$x = 196.53$

9. Incorrect statement among the following is :
(1) SO_2 act as oxidising agent but not reducing agent.
(2) NO_2 exist as dimer
(3) PF_5 exist but NF_5 does not
(4) PH_3 has lower proton affinity than NH_3

Ans. **(1)**

10. Two radioactive decays are

$A \xrightarrow{\lambda_1} \text{product}$ $\lambda_1 = 3\lambda_2$

$B \xrightarrow{\lambda_2} \text{product}$ $N_{A_0} = N_{B_0}$

Find ratio of $(N_A)_t$ and $(N_B)_t$ after one half life of A

Ans. **(4)**

Sol. Radioactive decays obeys 1st order kinetics

$$\frac{(N_B)_t}{(N_A)_t} = \frac{N_{B_0} e^{-\lambda_2 (t_{1/2})_A}}{N_{A_0} e^{-\lambda_1 (t_{1/2})_A}} \quad (N_{A_0} = N_{B_0})$$

$$= \frac{e^{-\lambda_2 \times \frac{\ln 2}{\lambda_1}}}{e^{-\lambda_1 \times \frac{\ln 2}{\lambda_1}}} = \frac{e^{-\ln 8}}{e^{-\ln 2}} \Rightarrow \frac{(N_A)_t}{(N_B)_t} = 4$$

11. Calculate the percentage by weight of S if 160 g of organic compound produce 466 g of $BaSO_4$.

Ans. **(40)**

Sol. $S \rightarrow BaSO_4$

(atoms of S)$_S$ = (atoms of S)$_{BaSO_4}$

$$\left(\frac{wt}{32}\right) \times N_A \times 1 = \left(\frac{466}{233}\right) \times N_A \times 1$$

$$(wt)_s = \left(\frac{466 \times 32}{233}\right)$$

$(wt)_s = 64$ g

$$\% S = \frac{(wt)_s}{(wt)_{org.\ compound}} \times 100 = \frac{64}{160} \times 100 = 40\ \%$$

12. Find the spectral line of H-atom, which have $\lambda = 900$ nm, $R_H = 10^5\ cm^{-1}$

(1) $n_2 = \infty \rightarrow n_1 = 1$, Lyman
(2) $n_2 = \infty \rightarrow n_1 = 2$, Balmer
(3) $n_2 = 5 \rightarrow n_1 = 3$, Paschan
(4) $n_2 = \infty \rightarrow n_1 = 3$, Paschan

Ans. **(4)**

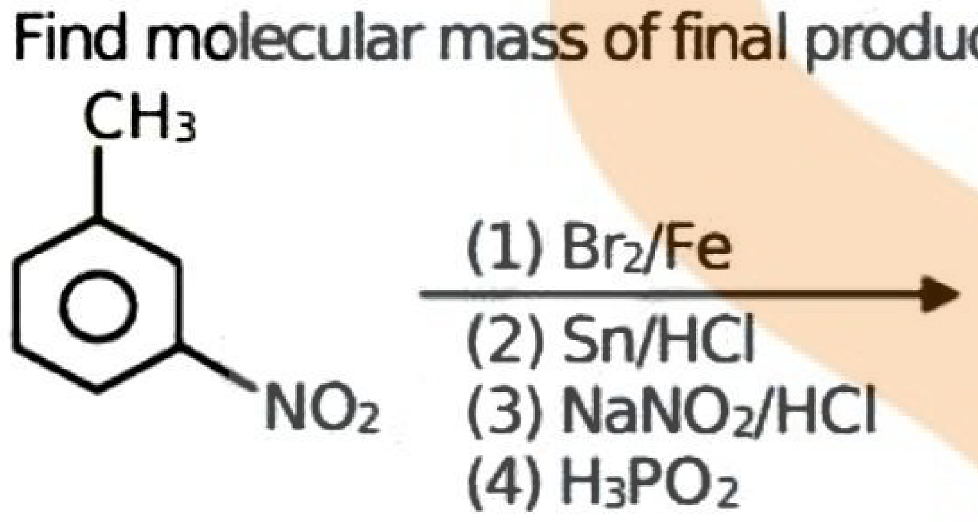
Sol. $$\frac{1}{\lambda} = R_H \times 2^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = 10^5 \times 1 \times \left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$$

$$\frac{1}{\lambda} = \frac{10^5}{9}$$

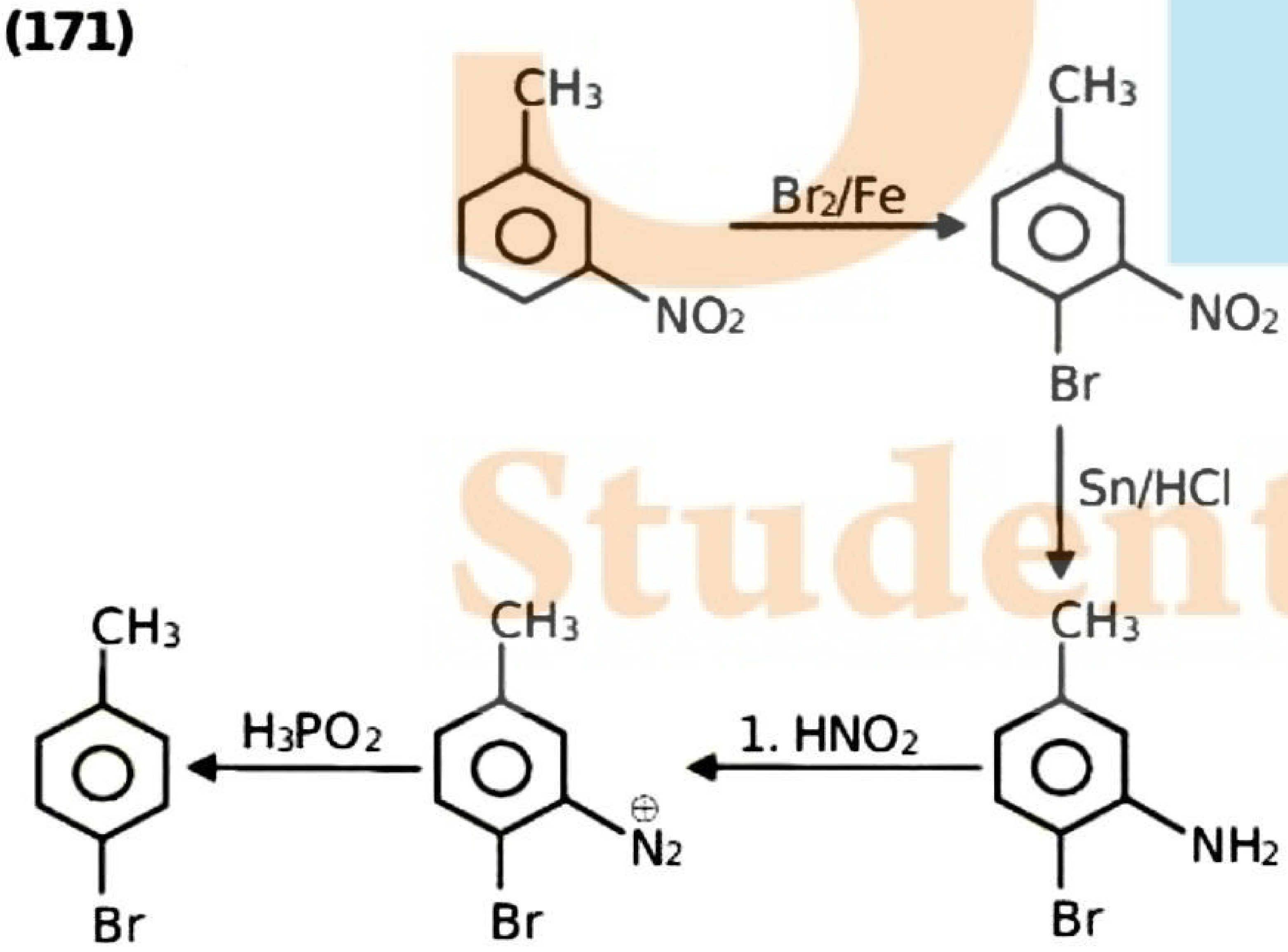
$\Rightarrow \quad \lambda = 9 \times 10^{-5}$ cm $= 900 \times 10^{-7}$ cm $= 900$ nm

13. Find molecular mass of final product.

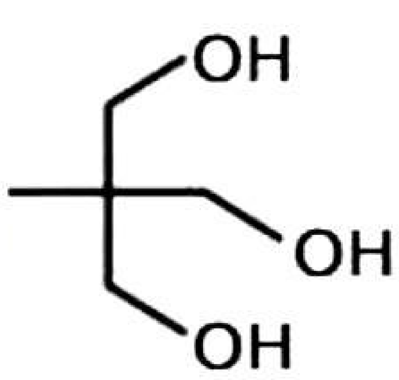
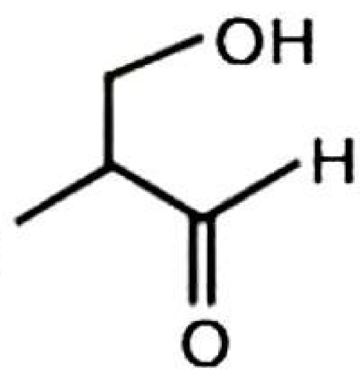
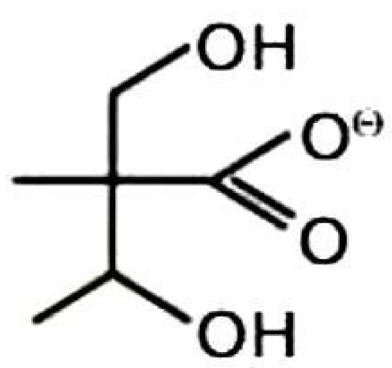
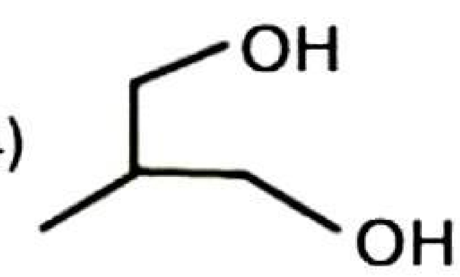


Ans. **(171)**

Sol.

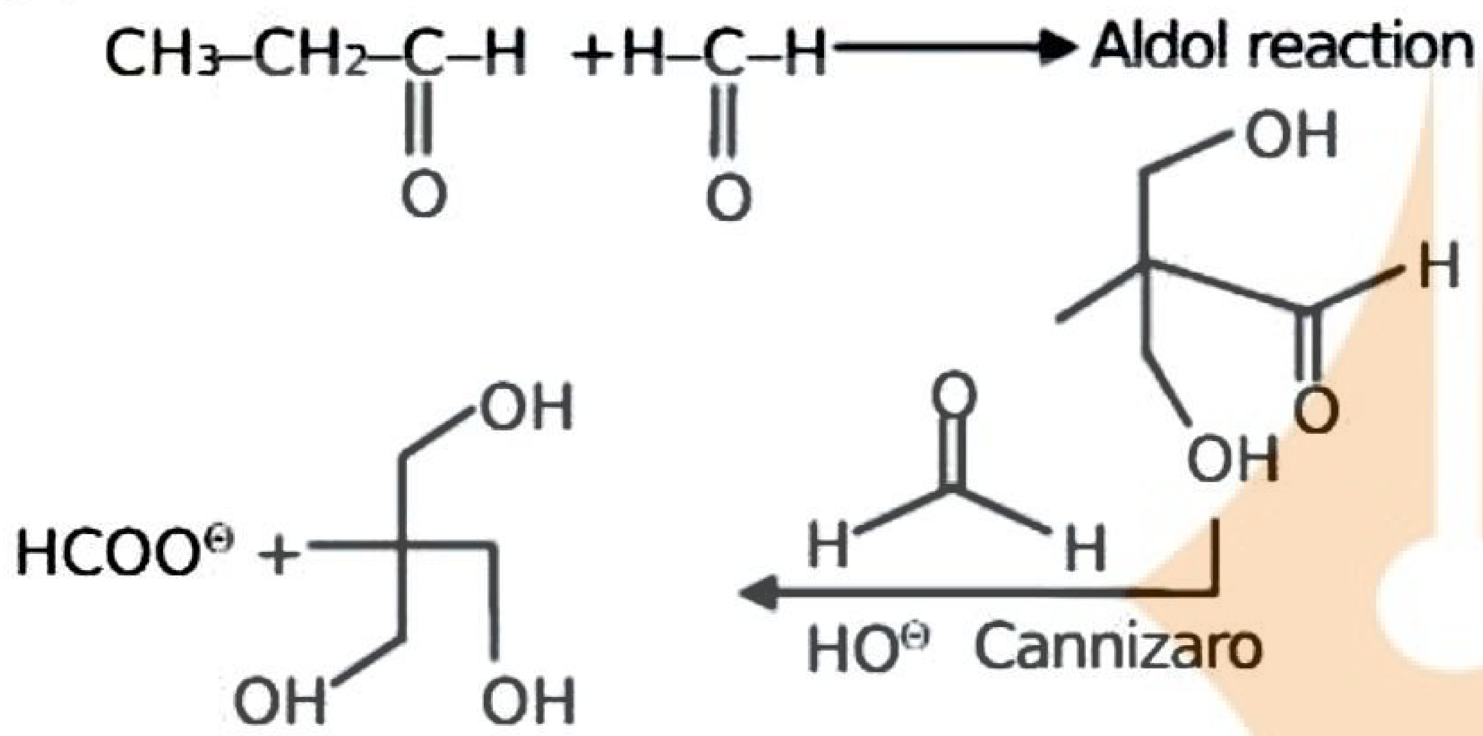


14. $CH_3-CH_2-\underset{\underset{O}{\|}}{C}-H + H-\underset{\underset{O}{\|}}{C}-H \xrightarrow{\text{Alkali reflux}}$
(Excess)

(1)  (2)  (3)  (4) 

Ans. **(1)**

Sol.



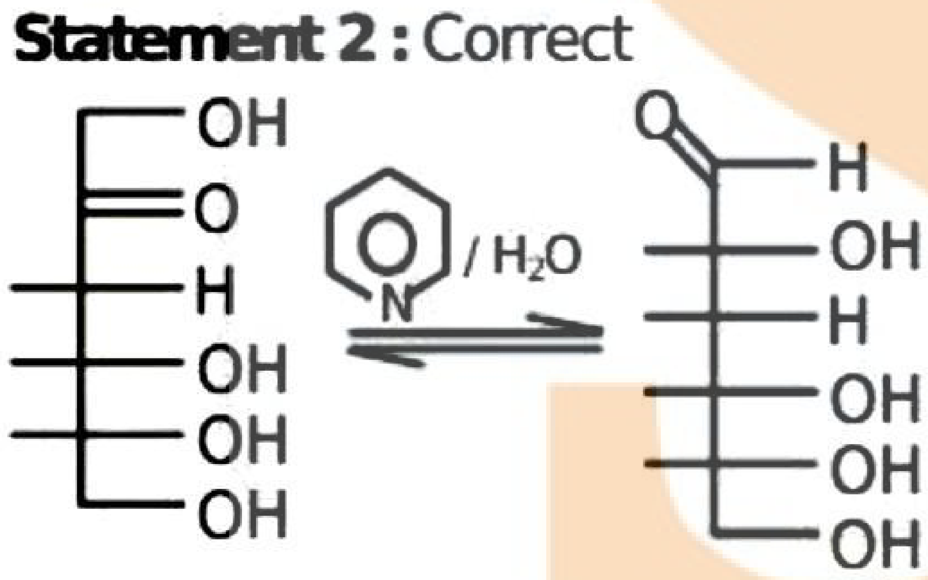
15. **Statement 1 :** Fructose gives silver mirror with Tollens reagent although $-\underset{\underset{O}{\|}}{C}-H$ group is absent in it.

Statement 2 : Fructose in Alkaline (Base) medium converts into Aldose Sugar Glucose which has $-\underset{\underset{O}{\|}}{C}-H$ group.

(1) Both Statement 1 and statement 2 are true
(2) Both statement 1 and statement 2 are false
(3) Statement 1 is true but statement 2 is false
(4) Statement 1 is false but statement 2 is true

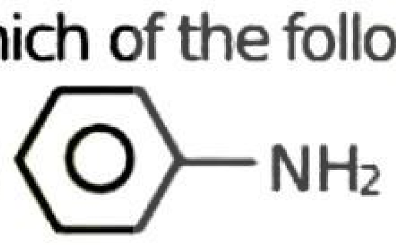
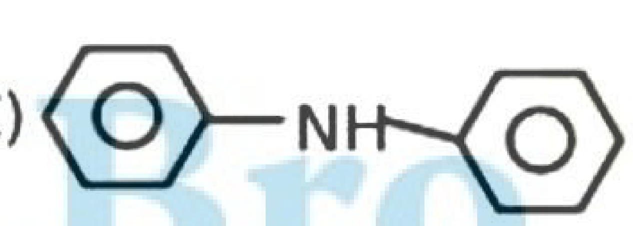
Ans. **(1)**

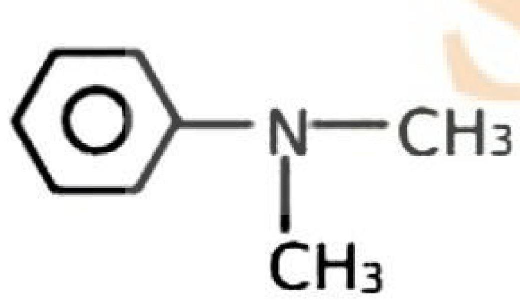
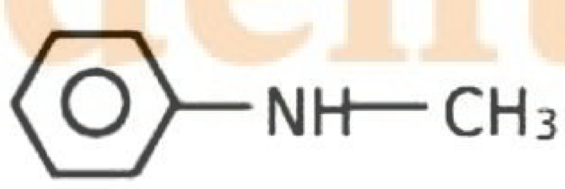
Sol. **Statement 1 :** Correct
Statement 2 : Correct



Rearrangement or inter conversion between fructose and glucose.

16. Which of the following reacts with Hinsberg reagent

(A)  (B) CH_3-NH_2 (C) 

(D)  (E) 

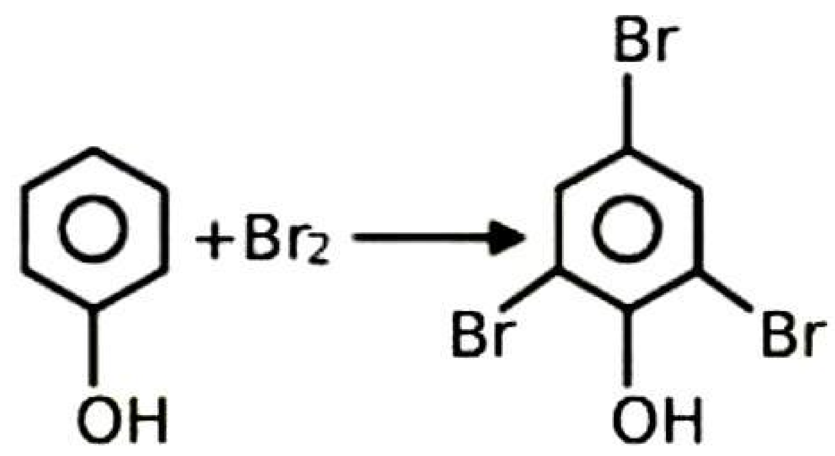
(1) A, B, C, E (2) B, C, D (3) A, C, D, E (4) C, D, E

Ans. **(1)**

Sol. Only primary and sec. amine reacts with Hinsberg reagent.

17. 2 g phenol react with Br_2 water to give trisubstituted phenol.
How much Br_2 is needed to complete reaction in grams. (Rounded off to nearest integer)

Ans. **(10)**

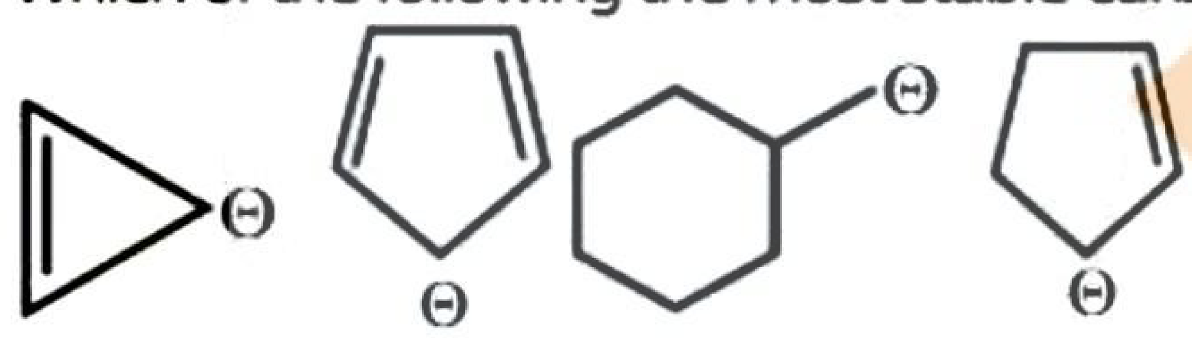
Sol. 

1 : 3 mol ratio

$\frac{2g}{94}$ $\quad \frac{2}{94} \times 3$ mol Br_2

Hence $\frac{2 \times 3}{94} \times 160g\,Br_2$ used in reaction. $=10.21$ g

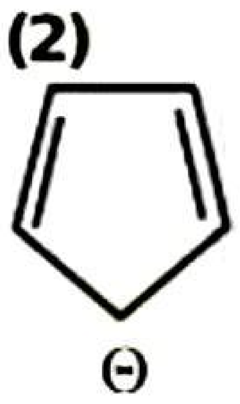
18 Which of the following the most stable carbanion is



P Q R S

(1) P (2) Q (3) R (4) S

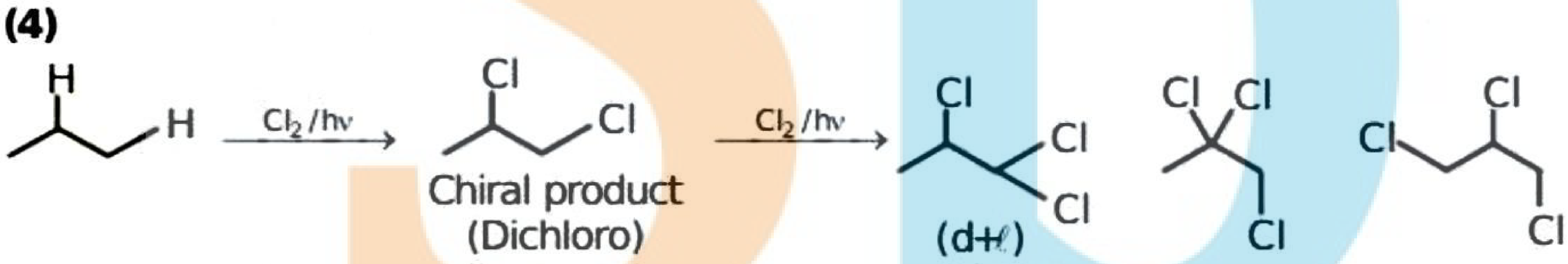
Ans. **(2)**

Sol.  Follows Huckel rule hence is Aromatic stabilised resonance energy.

19. Propane reacts with Cl_2 in sunlight to give chiral product $\boxed{x}$ which is dichloro product.

$\boxed{x}$ is further chlorinated in sunlight to give how many trichloro product.

Ans. **(4)**

Sol. 

Total 4 product are formed.

20. In estimation of sulphur by carius method, 160 g of organic compound gives 466 g of Barium sulphate. % of sulphur in the organic compound is.

Ans. **(40)**

Sol. $BaSO_4(233)$

Moles $= \frac{466}{233} = 2$

Mass(s) $= 2 \times 32 = 64$

% of S $= \frac{64}{160} \times 100 = 40\%$

21. Match the reactions name in given column to correct product formed.

	Column-I		**Column-II**
(A)	Wurtz fittig reaction	(P)	Fluoride product
(B)	Finkelstein Reaction	(Q)	Iodide product
(C)	Sand Meyer Reaction	(R)	Chloride product
(D)	Swart Reaction	(S)	Hydrocarbon product

(1) A-(S); (B)-(Q); (C)-(R); (D)-(P)

(2) A-(R); (B)-(P); (C)-(S); (D)-(P)

(3) A-(P); (B)-(R); (C)-(Q); (D)-(R)

(4) A-(Q); (B)-(R); (C)-(S); (D)-(P)

Ans. (1)

PART : MATHEMATICS

1. Find number of words by using all letters of the word "DAUGHTER" such that no two vowels come together

(1) 5200 (2) 7200 (3) 14400 (4) $3\times\lfloor 5$

Ans. **(3)**

Sol.

D	G	H	T	R

Number of ways of arrangement of consonants = $\lfloor 5$

Now there are 6 gaps between these consonants.

So, number of ways of arrangement of three vowels A, U, E = ${}^6P_3$

So total number of words = $\lfloor 5 \times {}^6P_3 = 120 \times 6 \times 5 \times 4 = 120 \times 120 = 14400$

2. Find sum of all rational terms in expansion of $(1 + 2^{1/3} + 3^{1/2})^6$

(1) 144 (2) 612 (3) 720 (4) 562

Ans. **(2)**

Sol. General term = $\dfrac{\lfloor 6}{\lfloor r_1 \lfloor r_2 \lfloor r_3} \times 2^{\frac{r_2}{3}} \times 3^{\frac{r_3}{2}}$; $0 \le r_1, r_2, r_3 \le 6$ and $r_1 + r_2 + r_3 = 6$.

For rational term : $r_2 = 0 \rightarrow r_1 + r_3 = 6 \begin{cases} r_3 = 0, r_1 = 6 \\ r_3 = 2, r_1 = 4 \\ r_3 = 4, r_1 = 2 \\ r_3 = 6, r_1 = 0 \end{cases}$

$r_2 = 3 \rightarrow r_1 + r_3 = 3 \begin{cases} r_3 = 0, r_1 = 3 \\ r_3 = 2, r_1 = 1 \end{cases}$

$r_2 = 6 \rightarrow r_1 + r_2 = 0 \quad (r_1 = 0, r_2 = 0$

Sum of all rational terms =

$$\frac{\lfloor 6}{\lfloor 0\lfloor 0\lfloor 6}2^0.3^0 + \frac{\lfloor 6}{\lfloor 0\lfloor 2\lfloor 4}2^0.3^1 + \frac{\lfloor 6}{\lfloor 0\lfloor 4\lfloor 2}2^0.3^2 + \frac{\lfloor 6}{\lfloor 0\lfloor 6\lfloor 0}2^0.3^3 + \frac{\lfloor 6}{\lfloor 3\lfloor 0\lfloor 3}2^1.3^0 + \frac{\lfloor 6}{\lfloor 3\lfloor 2\lfloor 1}2^1.3^1 + \frac{\lfloor 6}{\lfloor 6\lfloor 0\lfloor 0}2^2.3^0$$

$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$

3. If for an AP, if first term is 3 and sum of first four terms is equal to $\frac{1}{5}$ of the sum of next four terms, then the sum of first 20 terms is:

(1) –540 (2) –1080 (3) 2016 (4) 4080

Ans. **(2)**

Sol. a = 3, let common difference = d.

$\frac{4}{2}[2\times 3 + (4-1)d] = \frac{4}{5\times 2}[2\times(3+4d)+(4-1)d]$

$5(6 + 3d) = 6 + 8d + 3d$

$30 + 15d = 6 + 11d$

$4d = -24$

$d = -6$

$S_{20} = \frac{20}{2}[2\times 3 + 19(-6)] = 10[6 - 114] = 10 \times (-108) = -1080$

4. Value of $\sin 70^\circ(\cot 10^\circ \cot 70^\circ - 1)$ is:

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) 3

Ans. **(2)**

Sol. $\sin 70^\circ\left(\frac{\cos 10^\circ}{\sin 10^\circ}\frac{\cos 70^\circ}{\sin 70^\circ} - 1\right)$

$\sin 70^\circ\left(\frac{\cos 70^\circ \cos 10^\circ - \sin 70^\circ \sin 10^\circ}{\sin 70^\circ \sin 10^\circ}\right) = \frac{\cos(70^\circ + 10^\circ)}{\sin 10^\circ} = \frac{\cos 80^\circ}{\sin 10^\circ} = \frac{\sin 10^\circ}{\sin 10^\circ} = 1$

5. Value of $\cos^{-1}\left[\frac{12}{13}\cos x + \frac{5}{13}\sin x\right]$ is, if $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

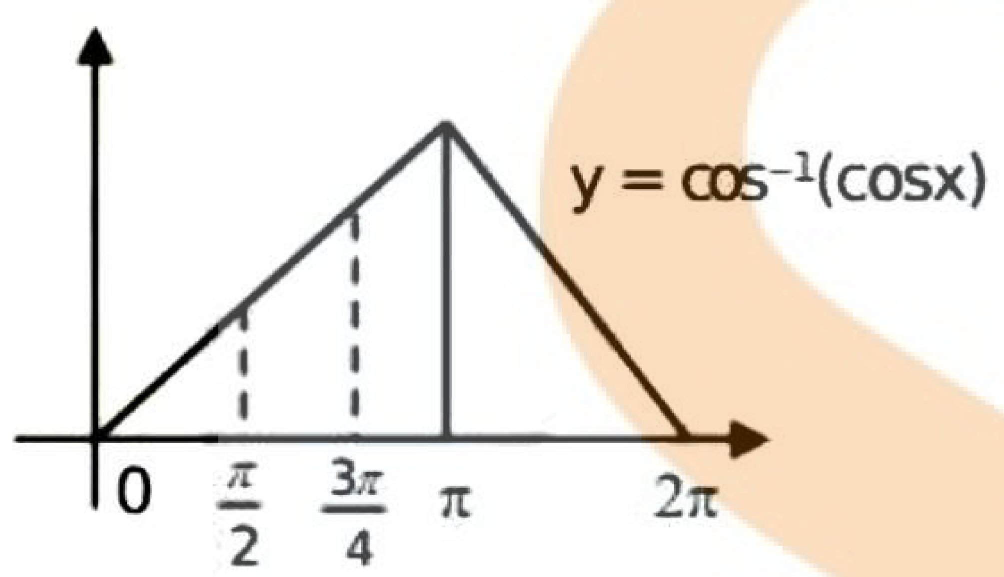
(1) $x + \tan^{-1}\frac{12}{5}$ (2) $x - \tan^{-1}\frac{12}{5}$ (3) $x - \tan^{-1}\frac{5}{12}$ (4) $x + \tan^{-1}\frac{5}{12}$

Ans. **(3)**

Sol. $\cos^{-1}\left[\frac{12}{13}\cos x + \frac{5}{13}\sin x\right]$

$\cos^{-1}[\cos(x - \phi)]$

Let $\cos\phi = \frac{12}{13}$ and $\sin\phi = \frac{5}{13}$, so $\tan\phi = \frac{5}{12}$



$= x - \phi$

$= x - \tan^{-1}\frac{5}{12}$

6. If function $f(x) = \begin{cases} \frac{2}{x}\{\sin(k_1 + 1)x + \sin(k_2 + 1)x\} & x < 0 \\ 4 & x = 0 \\ \frac{2}{x}\ell n\left(\frac{k_2 x + 1}{k_1 x + 1}\right) & x > 0 \end{cases}$ is continuous at $x = 0$, then value of $k_1^2 + k_2^2$ is equal to –

(1) 6 (2) 2 (3) 4 (4) 8

Ans. **(2)**

Sol. $f(0) = 4$

$$f(0^-) = \lim_{h\to 0}\frac{2\{\sin(k_1+1)h+\sin(k_2+1)h\}}{h}$$

$$= \lim_{h\to 0}\frac{2\{(k_1+1)\cos(k_1+1)h+(k_2+1)\cos(k_2+1)h\}}{1}$$

$= 2\ (k_1 + 1 + k_2 + 1) = 2\ (k_1 + k_2 + 2)$

$$f(0^+) = \lim_{h\to 0}\frac{2\{\ell n(k_2h+1)-\ell n(k_1h+1)\}}{h}$$

$$= \lim_{h\to 0}\frac{2\left\{\frac{k_2}{k_2h+1}-\frac{k_1}{k_1h+1}\right\}}{1}$$

$= 2\ (k_2 - k_1)$

Now $2\ (k_1 + k_2 + 2) = 4$

$k_1 + k_2 = 0$ ...(i)

and $2(k_2 - k_1) = 4$

$k_2 - k_1 = 2$ ...(ii)

So $k_1 = -1$ and $k_2 = 1$

So $k_1^2 + k_2^2 = 1 + 1 = 2$

7. A relation defined on set $A = \{1, 2, 3, 4\}$, then how many ordered pairs are added to $R = \{(1, 2), (2, 3), (3,3)\}$ so that it becomes equivalence?

Ans. **(7)**

Sol. For equivalence it must be transitive, symmetric and reflexive all.

For reflexive $\rightarrow$ (1, 1), (2, 2), (4, 4)

For symmetric $\rightarrow$ (2, 1), (3, 2)

For transitive $\rightarrow$ (1, 3), (3, 1)

Total 7 pairs has to be added to make it's an equivalence relation.

8. Find value of λ for which system of equation:

$(\lambda - 1)x + (\lambda + 2)y + (\lambda - 1)z = 0$

$\lambda x + (\lambda - 1)y + (\lambda + 1)z = 0$

$(\lambda - 1)x + (\lambda + 1)y + (\lambda + 2)z = 0$ has infinite solution.

(1) 0 (2) $\frac{2}{11}$ (3) 2 (4) $\frac{3}{11}$

Ans. **(2)**

Sol. Homogeneous system of equation:

$$\begin{vmatrix} (\lambda-1) & (\lambda+2) & \lambda-1 \\ \lambda & \lambda-1 & \lambda+1 \\ \lambda-1 & \lambda+1 & (\lambda+2) \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} \lambda-1 & \lambda+2 & \lambda-1 \\ 1 & -3 & 2 \\ 0 & -1 & 3 \end{vmatrix} = 0$$

$(\lambda - 1)(-9 + 2) - (\lambda + 2)(3 - 0) + (\lambda - 1)(-1 - 0) = 0.$

$-7(\lambda - 1) - 3(\lambda + 2) - (\lambda - 1) = 0.$

$-7\lambda + 7 - 3\lambda - 6 - \lambda + 1 = 0$

$\Rightarrow \quad \lambda = \dfrac{2}{11}$

9. There are two biased dice such that, for first dice two faces show 1, 2 faces show 2, one face show 3 and one face show 4. For second dice one face show 1, two faces show 2, one face show 3, and two faces show 4. Then find probability of getting sum 4 or 5, when dice are thrown together.

(1) $\dfrac{5}{9}$ (2) $\dfrac{4}{9}$ (3) $\dfrac{2}{9}$ (4) $\dfrac{8}{9}$

Ans. **(2)**

Sol. First dice have 1, 1, 2, 2, 3, 4

Second dice have 1, 2, 2, 3, 4, 4

Now P (sum 4 or sum 5) = P (sum 4) + P (sum 5)

= P (1, 3) + P (2, 2) + P (3, 1) + P (1, 4) + P(2, 3) + P(3, 2) + P(4, 1)

$= \dfrac{2}{6}\times\dfrac{1}{6} + \dfrac{2}{6}\times\dfrac{2}{6} + \dfrac{1}{6}\times\dfrac{1}{6} + \dfrac{2}{6}\times\dfrac{2}{6} + \dfrac{2}{6}\times\dfrac{1}{6} + \dfrac{1}{6}\times\dfrac{2}{6} + \dfrac{1}{6}\times\dfrac{1}{6}$

$= \dfrac{2+4+1+4+2+2+1}{36} = \dfrac{16}{36} = \dfrac{4}{9}$

10. If $\left|\dfrac{\bar{z}-i}{2\bar{z}+i}\right| = \dfrac{1}{3}$ represent a circle whose centre is C and area of triangle whose vertices are (0, 0), C and (α, 0) is 11 then find α^2.

Ans. **(100)**

Sol.

$3|\bar{z} - i| = |2\bar{z} + i|$

$3|x - i(y + 1)| = |2x + i(1 - 2y)|$

$\Rightarrow \quad 9(x^2 + (y + 1)^2) = 4x^2 + (1 - 2y)^2$

$\Rightarrow \quad 5x^2 + 5y^2 + 22y + 8 = 0$

Centre $C\left(0, \dfrac{-11}{5}\right)$

Area $\Delta = \left|\dfrac{1}{2}\cdot\dfrac{11}{5}\cdot\alpha\right| = 11$

$|\alpha| = 10$

$\alpha^2 = 100$

11. If both roots of quadratic equation

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal and $a + c = 15$, $b = 2/15$ then value of $a^2 + c^2$ is:

(1) 217 (2) 223 (3) 213 (4) 211

Ans. **(2)**

Sol. Clearly one root is one

$\therefore$ Product of roots $= 1$

$$\frac{c(a-b)}{a(b-c)} = 1$$

$ac - bc = ab - ac$

$2ac = b(a + c)$

$$2ac = \frac{2}{15} \times 15$$

$ac = 1$

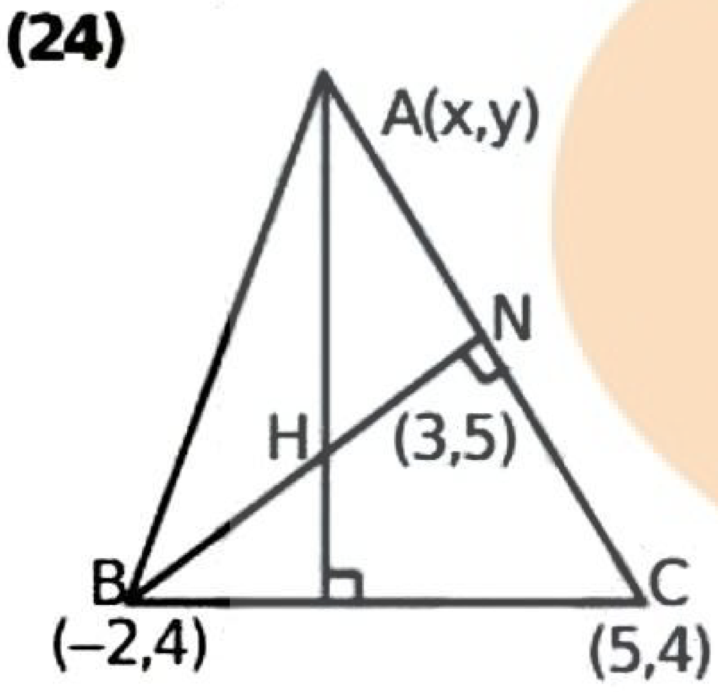
Now $a^2 + c^2 = (a + c)^2 - 2ac$

$= (15)^2 - 2 = 223$

12. Two vertices of triangle are $(-2, 4)$ and $(5, 4)$ and its orthocentre is $(3, 5)$ and centroid is (c, d) then the value of $c + 3d$ is:

Ans. **(24)**

Sol.



BC $\Rightarrow$ parallel to x – axis

AH $\Rightarrow$ parallel to y – axis

so $x = 3$

$M_{AC} \times M_{BN} \Rightarrow -1$

$$\frac{4-y}{5-3} \times \frac{5-4}{3+2} = -1$$

$= (4 - y) = -10$

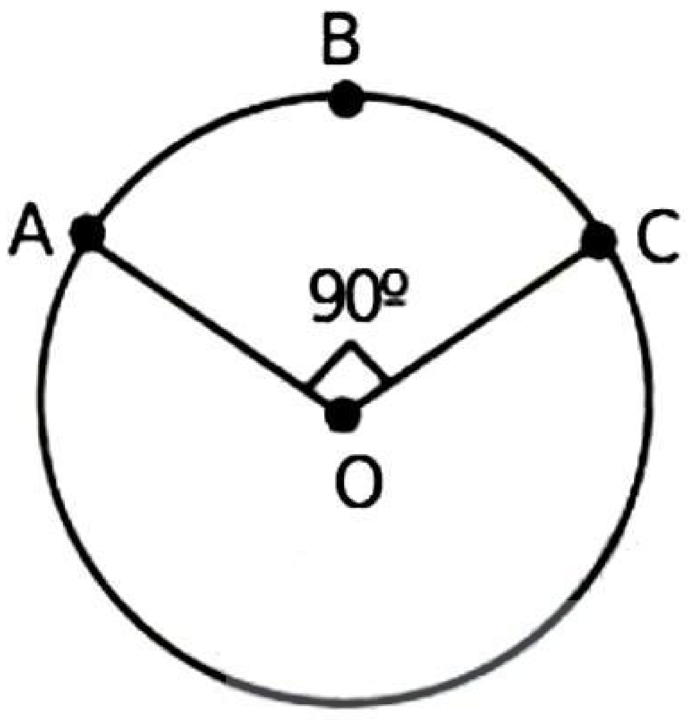
$\Rightarrow y = 14$

A (3,14)

$G \equiv (c, d) \equiv \left(2, \frac{22}{3}\right) \quad \Rightarrow \quad c = 2, d = \frac{22}{3}$

$c + 3d = 24$

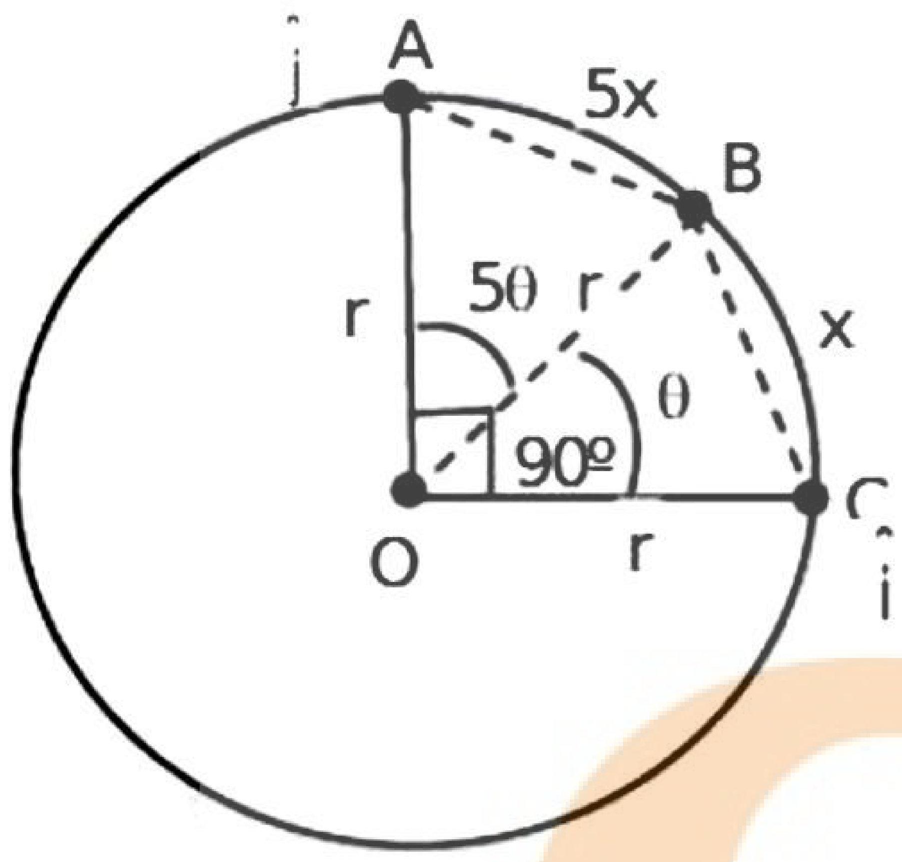
13. In the given figure, $\frac{AB}{BC}=\frac{1}{5}$, $\overrightarrow{OC}=\alpha\overrightarrow{OA}+\beta\overrightarrow{OB}$; find $\alpha+\sqrt{2}(\sqrt{3}-1)\beta$



(1) $(\sqrt{3}+1)\frac{3}{2\sqrt{2}}$ (2) $(\sqrt{3}-1)\frac{5}{2\sqrt{2}}$ (3) $(\sqrt{3}-1)\frac{3}{2\sqrt{2}}$ (4) $(\sqrt{3}+1)\frac{5}{2\sqrt{2}}$

Ans. **(3)**

Sol.



$5\theta+\theta=\frac{\pi}{2}$

$\theta=\frac{\pi}{12}$

$\overrightarrow{OC}=r\cos\theta\hat{i}$

$\overrightarrow{OA}=r\hat{j}$

$\overrightarrow{OB}=r\cos\theta\hat{i}+r\sin\theta\hat{j}$

$\overrightarrow{OC}=\alpha\overrightarrow{OA}+\beta\overrightarrow{OB}$

$r\cos\theta\hat{i}=\alpha(r\hat{j})+\beta(r\cos\theta\hat{i}+r\sin\theta\hat{j})$

$r\cos\theta=\beta r\cos\theta$

$\beta=1$

$\alpha r+r\sin\theta.\beta=0$

$\alpha=-\sin\theta$

$\alpha=-\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

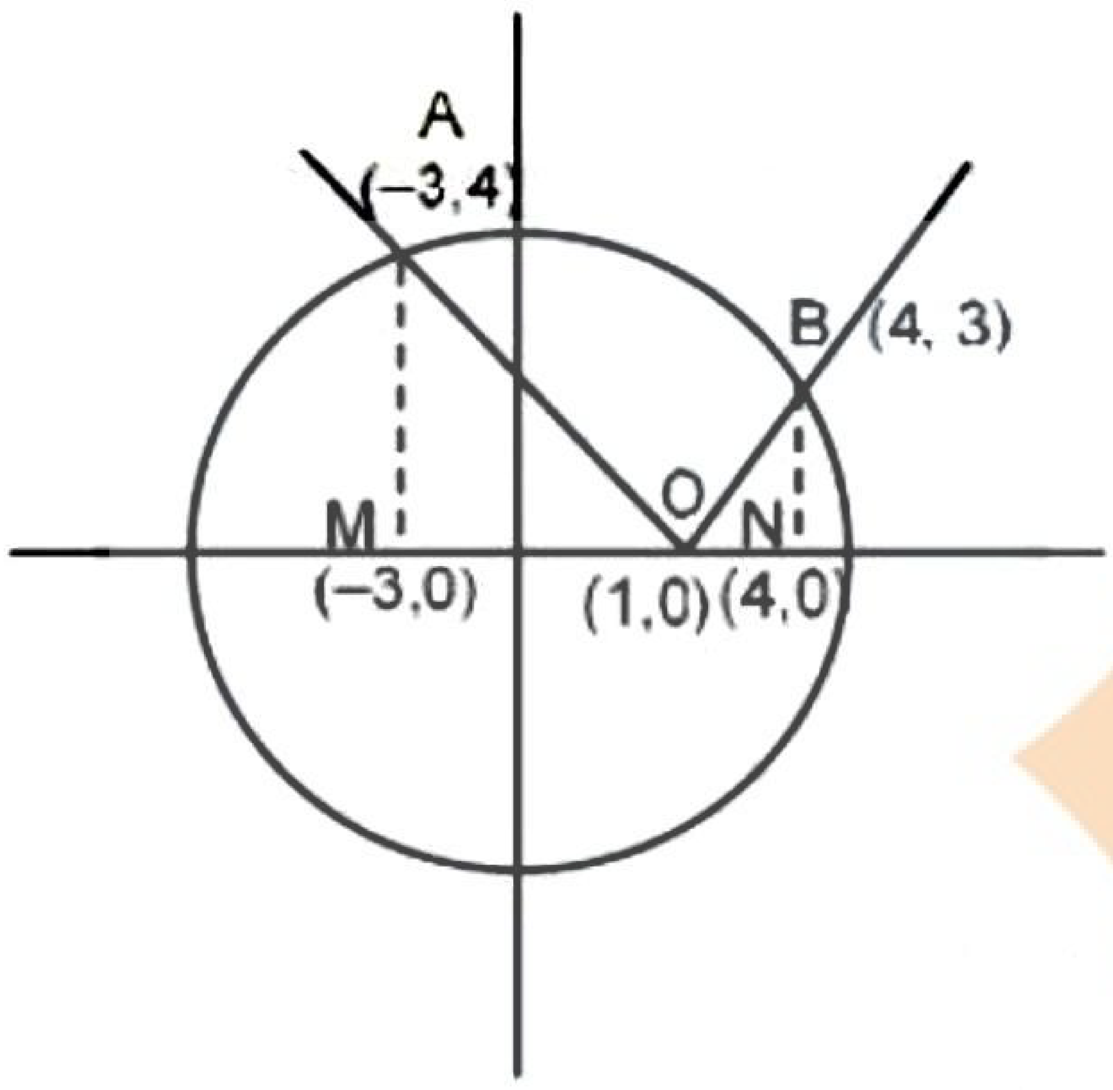
$-\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+\sqrt{2}(\sqrt{3}-1).1=(\sqrt{3}-1)\frac{3}{2\sqrt{2}}$

14. Area of the larger region bounded by curves $y = |x-1|$ and $x^2 + y^2 = 25$ is:

(1) $\left(\frac{75\pi}{4} + \frac{1}{2}\right)$ (2) $\left(\frac{75\pi}{4} - \frac{1}{2}\right)$ (3) $\left(\frac{25\pi}{4} + \frac{1}{2}\right)$ (4) $\left(\frac{25\pi}{4} - \frac{1}{2}\right)$

Ans. **(1)**

Sol. Area of shaded of region = area of circle – area AOB (unshaded)



$\therefore$ Area of AOB $= \int_{-3}^{4} \sqrt{25-x^2}$ – Area of ΔAOM – Area of ΔONB

$$= \int_{-3}^{4} \sqrt{25-x^2} - \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 3 \times 3$$

$$= \int_{-3}^{4} \sqrt{25-x^2} - 8 - \frac{9}{2} \quad \ldots(i)$$

$$= \left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4} - \frac{25}{2}$$

$$= \frac{25\pi}{4} - \frac{1}{2}$$

$\therefore$ Required area $= \pi(5^2) - \left(\frac{25\pi}{4} - \frac{1}{2}\right) = \frac{75\pi}{4} + \frac{1}{2}$

15. Given $f(x) = \ell nx$ and $g(x) = \dfrac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$, then the domain of f(g(x)) is :

(1) $(0, \infty)$ (2) $(1, \infty)$ (3) R (4) $(-\infty, 0)$

Ans. **(3)**

Sol. $f(g(x)) = \ell n(g(x)) \Rightarrow g(x) > 0$

$$g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1} > 0$$

$\Rightarrow$ $2x^2 - 2x + 1 > 0 \ \forall \ x \in R$ $(\because a > 0, D < 0)$

Now, $x^4 - 2x^3 + 3x^2 - 2x + 2 = x^4 - 2x^3 + 2x^2 + x^2 - 2x + 2$

$x^4 - 2x^3 + 2x^2 + x^2 - 2x + 2 = (x^2 + 1)(x^2 - 2x + 2) > 0 \ \forall \ x \in R$

$x^4 - 2x^3 + 3x^2 - 2x + 2 > 0$

$g(x) > 0 \ \forall \ x \in R$

Therefore, domain of f(g(x)) is R.